Journal of
Applied Crystallography
ISSN 0021-8898
Editor: **Gernot Kostorz**

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Journal of
Applied Crystallography
ISSN 0021-8898

Received 14 June 2007
Accepted 6 August 2007

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The statistical properties of intensities affected by counting loss based on conventional non-extended and extended dead-time models are examined by a Monte Carlo method. It has been confirmed that the variance of the counted pulses for the non-extended dead-time model with the rate of generated pulses r and the dead-time τ is given by $\sigma^2_{\rm non} = \mu_{\rm non}/(1+r\tau)^2$, while that for the extended dead-time model is given by $\sigma^2_{\rm ext} = \mu_{\rm ext}[1 - 2r\tau\exp(-r\tau)]$, as proposed by Laundy & Collins [(2003). *J. Synchrotron Rad.* **10**, 214–218], for the mean values of counted pulses $\mu_{\rm non}$ and $\mu_{\rm ext}$, respectively. Practical formulae to estimate the statistical errors of the corrected intensities are also presented.

1. Introduction

Counting methods are widely used to measure the intensity of X-rays. Independently generated signal pulses are expected to obey the Poisson distribution, which predicts that the possibility for the number of pulses n during the measurement period T is given by

$$P_{\rm Poisson}(n) = (n!)^{-1}(rT)^n\exp(-rT),$$

where r is the average rate of generated pulses. The mean and variance of the Poisson distribution are simply given by

$$\mu_{\rm Poisson} = \sigma^2_{\rm Poisson} = rT.$$

Therefore, the statistical errors of the intensity measured by a counting method can naturally be modelled by the square root of the measured count, if a Poisson distribution of the pulses is assumed.

However, it is not expected that the intensity measured with a realistic counting system should strictly obey the Poisson distribution, because it is always affected by the counting loss caused by the finite response time of a detector and/or the electronic circuits in the detection system.

The probability for the number of counted pulses m based on the non-extended dead-time model can be expressed as

$$P_{\rm non}(m) = (m!)^{-1}(1+r\tau)[r(T-m\tau)]^m\exp[-r(T-m\tau)], \qquad (1)$$

where τ is the dead-time. Details about the derivation and validation of the above formula will be discussed elsewhere. The mean and variance of the above distribution are approximately given by

$$\mu_{\rm non} = rT/(1+r\tau), \qquad (2)$$

$$\sigma^2_{\rm non} = \mu_{\rm non}/(1+r\tau)^2. \qquad (3)$$

Recently, Laundy & Collins (2003) have reported analytical formulae for the statistical properties of a pulse-overlap model for the counting loss, which includes the conventional extended dead-time model as a special case. According to their results, the mean and variance for the extended dead-time model are given by

$$\mu_{\rm ext} = rT\exp(-r\tau), \qquad (4)$$

$$\sigma^2_{\rm ext} = \mu_{\rm ext}[1-2r\tau\exp(-r\tau)]. \qquad (5)$$

It should be noted that the variance of the number of counted pulses is expected to be smaller than the mean value in both the non-extended and the extended dead-time models, while it should coincide with the mean in the case of the Poisson distribution.

Since all the formulae given in equations (2)–(5) are derived as the solution for the limit $T \gg \tau$, it will be worth examining the validity of the application to the case of a finite ratio of T to τ.

In this communication, the statistics of the non-extended and extended dead-time models are examined by applying a Monte Carlo method with a simple algorithm, and compared with the theoretical values calculated from equations (2)–(5). The statistical errors in the corrected intensity estimated by the conventional models of counting loss are also discussed.

2. Simulation

A simulation of conventional models for counting loss applied to pulses that obey the Poisson distribution is easily implemented as follows: (i) a pulse is generated at the time $t = t_0 + \Delta t$ after a random interval Δt from the time t_0 when the last pulse is generated, where Δt is calculated by $\Delta t = -(\ln x)/r$ from a random number x evenly distributed between 0 and 1; (ii) the pulse is counted when $t - t_0 > \tau$ in the case of the extended dead-time, and $t - t_{\rm v} > \tau$, where $t_{\rm v}$ is the arrival time of the last counted pulse, in the case of the non-extended dead-time model; (iii) t_0 is replaced by t, and $t_{\rm v}$ is also replaced by t when the pulse is counted. Processes (i)–(iii) are repeated while $t < T$.

Fig. 1 plots the mean and variance of 1000 trials of the non-extended dead-time simulations for various rates of pulse r. The measurement period and the dead-time are fixed at $T = 1$ and $\tau = 0.001$. The results of the simulation are well modelled by the curves given by equations (2) and (3) up to $r \simeq \tau^{-1}$.

The corrected intensity of each non-extended dead-time simulation is calculated by

$$c_i = m_i/(1 - m_i\tau/T), \qquad (6)$$

where m_i is the number of counted pulses. The variance of the corrected intensity $\{c_i\}$ and the corresponding variance curve derived from equation (2), given by

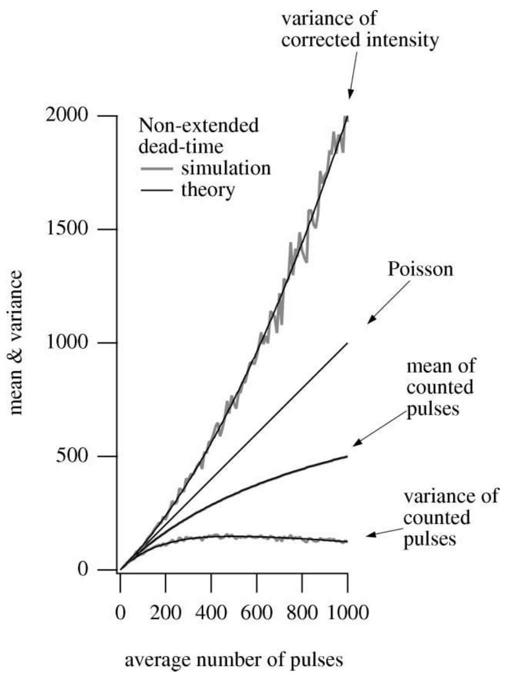

Figure 1
Mean and variance of the non-extended dead-time simulation and the theoretical dependence on variation of the average rate of pulses obeying Poisson statistics.

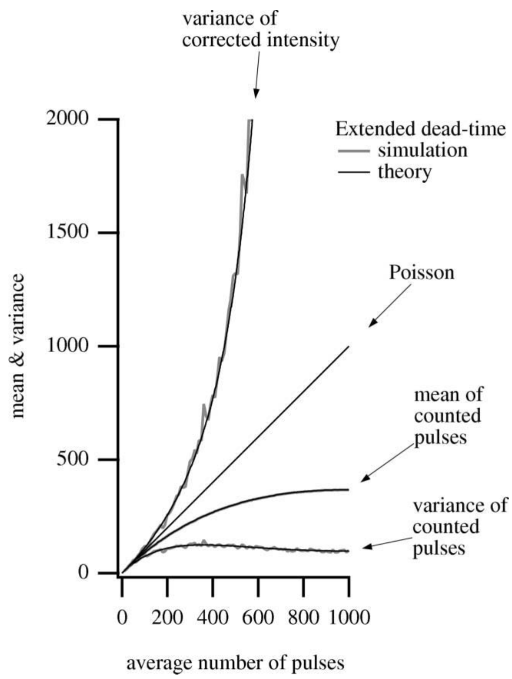

Figure 2
Mean and variance of the extended dead-time simulation and the theoretical dependence on variation of the average rate of pulses obeying Poisson statistics.

$$\sigma^2 = rT(1 + r\tau), \qquad (7)$$

are also plotted in Fig. 1. Even though the simulated statistical errors of the corrected intensity rapidly increase when the rate of pulse r approaches τ^{-1}, the errors are reasonably modelled by applying equation (7).

Fig. 2 plots the results for the extended dead-time simulations and corresponding theoretical curves. The parameters used in the extended dead-time simulation are the same as those for the non-extended dead-time simulation. The results of simulation are well modelled by the theoretical curves calculated by equations (4) and (5), again up to $r \simeq \tau^{-1}$.

The corrected intensity data $\{c_i\}$ for the extended dead-time simulation are calculated by applying the following approximation (Ida & Iwata, 2005):

$$c_i = r''T/(1 - r''t_1),$$

$$r'' = -t_2^{-1} \ln\{[1 + (1 - 4m_i t_2/T)^{1/2}]/2\},$$

$$t_1 = [1 - 3(6/13)^{1/2}/2]\tau,$$

$$t_2 = (6/13)^{1/2}\tau.$$

The variance of the corrected intensity from the extended dead-time simulation is well modelled by the variance curve given by (Laundy & Collins, 2003)

$$\sigma^2 = rT(1 - r\tau)^{-2}[\exp(r\tau) - 2r\tau], \qquad (8)$$

as shown in Fig. 2.

In conclusion, the statistical errors of the corrected intensity, which should properly be taken into account in least-squares or maximum-likelihood analysis (Antoniadis & Berruyer, 1990), can be estimated by applying equation (7) or (8).

References

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